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Price Forecasting with State-Space Models of Nonstationary Time Series: Case of the Japanese Salmon Market

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Abstract—The paper compares four state-space methods for modeling nonstationary vector-valued time series. An estimation of covariance model parameters based on the Kung-Vaccaro method is introduced, and the least squares solution to the vector of initial states is derived. Assuming stochastic nonstationarities, a two-step state-space procedure based on the cointegration representation is compared to the procedure based on the error correction representation. Assuming deterministic nonstationarities, the method of adding a regression equation to the innovations model is compared to a new technique that replaces the structural model by an impulse response model. The procedures are compared by generating short term forecasts of salmon wholesale prices in Japan.

Keywords—Nonstationarities, Error correction, Cointegration, Impulse response models.

1. INTRODUCTION

Based on the insights of linear systems theory, a state-space method for modeling vector-valued time series has been proposed by Aoki [1,2]. As is typical of most time series procedures, the method assumes that series are stationary stochastic processes. In particular, weak stationarity is assumed which requires mean, variance, and covariances of the series to be invariant with respect to displacement in time. Many time series in business and economics are not generated by stationary processes. In this case, accurate modeling of time series requires various transformations to achieve stationarity before the standard methods can be implemented.

Nonstationarities are divided into stochastic and deterministic. A modification of the standard state-space procedure based on the notion of cointegration is shown to be appropriate for stochastic trend removal, while the addition of a regression equation to the state-space model provides a method of accounting for deterministic trends (see [3]). In both cases, strict hypothesis of nonstationarity is abandoned in favor of separating dynamics into slow and fast modes.

In this paper, we experiment with two additional methods of modeling nonstationary vector-valued time series. The first one is the two step state-space procedure based on error correction representation suggested by Aoki and Havenner [4], and the second one is a new procedure in

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which deterministic components of the time series are modeled with impulse response model in the first step, and the resulting errors with innovations model in the second step.

The standard Aoki-Havenner method for estimating the parameters of covariance state-space model is replaced by an alternative method based on [5, pp. 705–714], and [6, pp. 105–122]. Additionally, the least squares solution to the vector of initial states is developed to replace the standard backcasting.

The procedures are compared by generating short term forecasts of salmon wholesale prices in Japan.

2. STATE-SPACE MODELING OF STATIONARY TIME SERIES

The state-space method for time series modeling is a stochastic identification procedure which requires obtaining a model for auto-covariance sequence of the process from raw data first, and then obtaining a model for stochastic process itself. The problem is one of fitting an approximate model to T observations on m series of a zero-mean, weakly stationary vector-valued Gaussian stochastic process $\{y_t\}$ with covariance sequence: $\Gamma_j = E[y_{t+j}\, y_t']$. It is assumed that y_t is a finite-dimensional process in the sense that it can be realized by passing the white noise through a finite dimensional linear system which consists of state equation and observation equation:

$$\begin{aligned} x_{t+1|t} &= Ax_{t|t-1} + Be_t, \\ y_t &= Cx_{t|t-1} + e_t. \end{aligned} \tag{1}$$

Input $\{e_t\}$ is a white Gaussian process with: $E(e_t\, e_s') = \delta_{ts}\, \Psi$, where δ_{ts} is Kronecker's delta defined as $\delta_{ts} = 1$ if $t = s$, or $\delta_{ts} = 0$, otherwise. The stochastic model in (1) is called an innovations model, where x_t is a vector of unobservable states that are minimal sufficient statistics for history of the process $\{y_t\}$. Subscripts on x refer to the conditional expectation of x in the period of the first subscript given the information set at the time of the second subscript. Matrices A, B, and C are the coefficients to be estimated.

The first step of the stochastic identification procedure is the estimation of the covariance model $\Gamma_j = CA^{j-1}\,\Omega$, for $j = 1, 2, \dots$, where $\Omega = E(x_{t+1}\, y_t')$ is the covariance of states with observations, and matrices A and C are identical to those in (1). In the second step, obtaining a stochastic model from a given covariance model requires an estimation of the Kalman filter matrix B. However, the method fails for all cases where the Fourier transform of the sequence defined by the covariance model is not positive semi-definite. The details of the estimation procedure and the solution to the positivity problem are given in [7].

2.1. Kung-Vaccaro Method of Estimating Covariance Model Parameters

Estimation of the covariance model parameters begins with specification of the number of past lags, k. As a practical matter, this depends on trading off sampling error in estimated auto-covariances at long lags against risk of not capturing all important lags. In the Aoki-Havenner procedure, the maximally distant auto-covariance lag to be modeled is $2k - 1$, which leads to the following Hankel matrix:

$$\hat{H} = \begin{pmatrix} \hat{\Gamma}_1 & \hat{\Gamma}_2 & \dots & \hat{\Gamma}_k \\ \hat{\Gamma}_2 & \hat{\Gamma}_3 & \dots & \hat{\Gamma}_{k+1} \\ \vdots & & & \\ \hat{\Gamma}_k & \hat{\Gamma}_{k+1} & \dots & \hat{\Gamma}_{2k-1} \end{pmatrix}, \tag{2}$$

where the $(m \times m)$ dimensional blocks Γ represent sample estimates of lag j auto-covariance matrices. The rank of the Hankel matrix (2) determines the order of the model $(\tilde{n})$. A computationally accurate method of determining the rank of the Hankel matrix is provided by the singular value decomposition (SVD). The essence of the procedure is to approximate the space spanned by the columns of the Hankel matrix of T observations on time series $\{y_t\}$ by the subspace spanned by the left singular vectors associated with significant singular values. The truncated Hankel matrix has the following SVD:

$$\hat{H}_{\tilde{n}} = \hat{U}_{\tilde{n}}\,\hat{\Sigma}_{\tilde{n}}\,\hat{V}'_{\tilde{n}}, \tag{3}$$

where the subscripts indicate that only the largest $\tilde{n}$ singular values and associated singular vectors are included in the approximation, $\Sigma_{\tilde{n}}$ is a diagonal matrix of singular values arranged in the descending order, and orthonormal columns of $U_{\tilde{n}}$ and $V_{\tilde{n}}$ are the left and the right singular vectors of $H_{\tilde{n}}$, respectively.

Recognition of the pattern generated by the covariance model allows another factorization of the Hankel matrix into its observability O, and controllability matrix K:

$$\hat{H}_{\tilde{n}} = \hat{O}_{\tilde{n}}\,\hat{K}_{\tilde{n}} = \begin{pmatrix} C \\ CA \\ \vdots \\ CA^{k-1} \end{pmatrix}_{\tilde{n}} \left(\Omega\, A\Omega\, A^2\Omega \ldots A^{k-1}\Omega\right)_{\tilde{n}}. \tag{4}$$

Estimation of the transition matrix A in the Aoki-Havenner procedure requires a time-shifted Hankel matrix for which a Γ_{2k} auto-covariance matrix is needed:

$$\overleftarrow{H} = \begin{pmatrix} \hat{\Gamma}_2 & \hat{\Gamma}_3 & \ldots & \hat{\Gamma}_{k+1} \\ \hat{\Gamma}_3 & \hat{\Gamma}_4 & \ldots & \hat{\Gamma}_{k+2} \\ \vdots & & & \\ \hat{\Gamma}_{k+1} & \hat{\Gamma}_{k+2} & \ldots & \hat{\Gamma}_{2k} \end{pmatrix}. \tag{5}$$

However, the entire matrix (5) is never used. The matrix actually shifted to the left is the truncated matrix $\hat{H}_{\tilde{n}}$ in (3), and the last block column of (5) is then used to replace the remaining void block column of $\hat{H}_{\tilde{n}}$. The drawback of this procedure is the fact that SVD is performed only on the Hankel matrix containing $(2k-1)$ auto-covariance lags, while $2k$ lags are actually available.

In the alternative procedure suggested here, the SVD is performed on the Hankel matrix with total available number of auto-covariance lags $(2k)$. Adding an extra block row, the modified Hankel matrix, which is now no longer a square matrix, becomes:

$$\hat{H} = \begin{pmatrix} \hat{\Gamma}_1 & \hat{\Gamma}_2 & \ldots & \hat{\Gamma}_k \\ \hat{\Gamma}_2 & \hat{\Gamma}_3 & \ldots & \hat{\Gamma}_{k+1} \\ \vdots & & & \\ \hat{\Gamma}_k & \hat{\Gamma}_{k+1} & \ldots & \hat{\Gamma}_{2k-1} \\ \hat{\Gamma}_{k+1} & \hat{\Gamma}_{k+2} & \ldots & \hat{\Gamma}_{2k} \end{pmatrix}. \tag{6}$$

Estimation of the system matrices A and C is based on the following property of the observability matrix: $OA = O\uparrow$, where $\uparrow$ indicates an upward shift by one block row:

$$\begin{pmatrix} C \\ CA \\ \vdots \\ CA^k \end{pmatrix} (A) = \begin{pmatrix} CA \\ CA^2 \\ \vdots \\ CA^{k+1} \end{pmatrix}. \tag{7}$$

Two factorizations of the Hankel matrix (6) based on (3) and (4) are equated to give:

$$\begin{aligned} \hat{H}_{\tilde{n}} &= \hat{U}_{\tilde{n}}\,\hat{\Sigma}_{\tilde{n}}^{1/2}\,\hat{\Sigma}_{\tilde{n}}^{1/2}\,\hat{V}'_{\tilde{n}} = \hat{O}_{\tilde{n}}\,\hat{K}_{\tilde{n}}, \\ \hat{O}_{\tilde{n}} &= \hat{U}_{\tilde{n}}\,\hat{\Sigma}_{\tilde{n}}^{1/2}. \end{aligned} \tag{8}$$

The observability matrix $\hat{O}_{\tilde{n}}$ is then partitioned into two submatrices: U_1 containing the first $(k \times m)$ rows, i.e., from row 1 to $(k \times m)$, and U_2 containing the last $(k \times m)$ rows, i.e., from row $(m+1)$ to $((k+1)\times m)$. Both U_1 and U_2 have $\tilde{n}$ columns corresponding to the determined number of states.

By simple inspection, the estimate of matrix C is simply the first block row of U_1 (or $\hat{O}_{\tilde{n}}$). The estimate of A is obtained by rewriting (7) as $U_1 A \approx U_2$ and solving this over-determined system in the least square sense:

$$\hat{A} = (U_1' U_1)^{-1} U_1' U_2. \tag{9}$$

The same estimation procedure would hold had an extra block column been added to the original Hankel matrix. Having an additional block row, however, enables the estimation of the matrix A with as little as $k=1$. Adding an extra block column to the original Hankel matrix requires at least $k=2$ to make the partitioning of the observability matrix $\hat{O}_{\tilde{n}}$ into U_1 and U_2 still possible.

To estimate Ω, notice that the final term in the auto-covariance sequence that forms the Hankel matrix (6) is $\Gamma_{2k} = CA^{2k-1}\,\Omega$. Having this in mind, the following relation holds:

$$\begin{pmatrix} C \\ CA \\ \vdots \\ CA^{2k-1} \end{pmatrix} (\Omega) \approx \begin{pmatrix} \hat{\Gamma}_1 \\ \hat{\Gamma}_2 \\ \vdots \\ \hat{\Gamma}_{2k} \end{pmatrix}. \tag{10}$$

The first matrix on the left-hand side of (10) is also an observability matrix but with different dimension than the one in (8). The observability matrix in (8) has $(k+1)$ block rows, while the one in (10) has $2k$ block rows, i.e., it has the dimension $((2k\times m)\times \tilde{n})$. The matrix on the right-hand side of (10) consists of all sample estimates of various lags auto-covariance matrices that form the Hankel matrix (6) stacked vertically. Denoting the observability matrix in (10) by Φ and the matrix of auto-covariances by R, the system can be solved in the least square sense to give the estimate of:

$$\hat{\Omega} = (\Phi' \Phi)^{-1} \Phi' R. \tag{11}$$

2.2. Estimating the Vector of Initial States

In order to generate forecasts based on the state-space model parameter estimates, an initial value of x_t has to be estimated. The least-square solution for the vector of initial states is derived as follows. First, substitution of the observation equation into the state equation is needed to get

$$x_{t+1} = (A - BC)\,x_t + By_t. \tag{12}$$

Equation (12) is then solved forward in time, and upon sequential substitution one gets

$$x_{t+k} = (A-BC)^k x_t + \sum_{j=0}^{k-1} (A-BC)^{k-1-j} By_{t+j}, \qquad k = 1, 2, \ldots. \tag{13}$$

By setting the initial state equal to zero, and making use of the fact that $E(y_{t+k}) = \hat{y}_{t+k} = Cx_{t+k}$, equation (13) becomes

$$\hat{y}_{t+k} = C \sum_{j=0}^{k-1} (A-BC)^{k-1-j} By_{t+j}. \tag{14}$$

From the definition of the k-step-ahead forecast error $\hat{e}_{t+k} = y_{t+k} - \hat{y}_{t+k}$, and upon the substitution of the observation equation from (1), together with (13) and (14), one gets

$$e_{t+k} = \hat{e}_{t+k} - C(A-BC)^k x_t, \qquad k = 0, 1, \ldots, T-1. \tag{15}$$

The idea is to minimize the expression (15) with respect to x_t in the least square sense. Constructing the matrix E from all $\hat{e}_{t+k}$, and the matrix Θ from all $C(A-BC)^k$, for $k = 0, 1, \ldots, T-1$, the matrix form of the function to be minimized can be written as

$$S = (E - \Theta\, x_t)'\,(E - \Theta\, x_t), \tag{16}$$

where E has dimension $((T \times m) \times 1)$, and Θ has dimension $((T \times m) \times \tilde{n})$. By setting $t = 1$ and taking the derivative of S with respect to x_1, and solving the first order condition for the extremum gives the estimate of the initial state x_1,

$$\hat{x}_1 = (\Theta' \Theta)^{-1} \Theta' E, \tag{17}$$

where

$$E = \begin{pmatrix} \hat{e}_1 \\ \hat{e}_2 \\ \hat{e}_3 \\ \vdots \\ \hat{e}_T \end{pmatrix}, \quad \Theta = \begin{pmatrix} C \\ C(A - BC) \\ C(A - BC)^2 \\ \vdots \\ C(A - BC)^{T-1} \end{pmatrix}. \tag{18}$$

Since it satisfies the least squares optimality criterion, the solution (17) is superior to the traditional backasting method, and should generally give better forecasting results.

3. NONSTATIONARITIES

Formally, a time series random variable is said to be stationary if its distribution does not depend on time. Conventionally, stationary variables are those with finite variances and autocovariances. Nonstationarities are categorized as stochastic and deterministic. Integrated stochastic processes, such as random walks or random walks with drift, exhibit stochastic nonstationarities. Processes depending on deterministic exogenous determinants, such as time trends or dummy variables for particular occurrences, exhibit deterministic nonstationarities. In this paper, we compare the forecasting performance of four procedures for modeling vector-valued nonstationary time series. Assuming nonstationarities are stochastic in nature, a two-step state-space procedure based on the notion of cointegration from [3] is compared to the modified procedure based on error correction specification from [4]. Assuming deterministic nonstationarities, the method of augmenting the innovations model with a regression equation from [8] is compared to the method where deterministic components are modeled as a response of the linear time-invariant discrete system to the Kronecker delta function.

3.1. Stochastic Nonstationarities

Time series usually contain both slowly-changing components (trends) and rapidly-changing components (cycles). When these components are of drastically different frequencies, successful modeling of stochastic processes may require partitioning the dynamic responses (eigenvalues of the dynamics matrix A) into two mutually exclusive sets. One containing the larger eigenvalues (closer to unity) corresponds to slower dynamics, and another containing smaller eigenvalues (closer to zero) corresponds to faster dynamics. Any decomposition into long-run and short-run dynamics is necessarily dependent on a particular set of restrictions, because separate data on trends and cycles are not observed. Those used here lead to decompositions based on the concepts of cointegration and error correction. The details about analytic decompositions of dynamic models can be found in [2, Chapters 3,11; 4; 9].

The rationale behind the concept of cointegration is that there exists a long-run equilibrium relationship among certain variables. In the short run, they may deviate from each other, but economic forces will bring them back together. The time series y_t are said to be cointegrated with cointegrating vector α if each element of a vector y_t achieves stationarity only after differencing, but a linear combination $\alpha' y_t$ is already stationary. Interpreting $\alpha' y_t = 0$ as a long run equilibrium, cointegration implies that deviations from equilibrium are stationary, with finite variance, even though the series themselves are nonstationary and have infinite variance.

The idea behind the error correction models is that a proportion of the disequilibrium from one period is corrected in the next period. For example, the change in price in one period may depend upon the degree of excess demand in the previous period. Such models usually describe optimal behavior with some types of adjustment costs or incomplete information. By the Granger Representation Theorem [10], the cointegrated series can be represented by error correction models, which means that cointegration implies and is implied by an error correction representation. Cointegration describes the long run equilibrium relationship between variables, and the error correction mechanism forces the short run deviations from equilibrium in one period to move towards the equilibrium in the next period.

The cointegration and error correction based state-space modeling procedures differ only in decomposition restrictions. The cointegration based procedure suggests that more accurate modeling of short cycles might be achieved if the long movements could be previously removed. This implies that cycle realization affects random trend, making trend model dependent on cycle model, but not vice-versa. The vector of observed series y_t is decomposed into long run dynamics summarized in a set of states $\tau_{t|t-1}$, and a stationary transformation of y_t, denoted by y_t^*, retaining the higher frequency dynamics:

$$\begin{aligned} \tau_{t+1|t} &= A_\tau \, \tau_{t|t-1} + B_\tau \, y_t^*, \\ y_t &= C_\tau \, \tau_{t|t-1} + y_t^*. \end{aligned} \tag{19}$$

In the second step, another state-space model designed to capture the remaining high frequency cycles is estimated:

$$\begin{aligned} \eta_{t+1|t} &= A_\eta \, \eta_{t|t-1} + B_\eta \, e_t, \\ y_t^* &= C_\eta \, \eta_{t|t-1} + e_t, \end{aligned} \tag{20}$$

where input into the trend model (19) y_t^* becomes the output of the cycle model (20), and e_t is a white Gaussian noise. Since short lags focus on long run dynamics, the number of auto-covariance lags in the trend model's Hankel matrix is usually set to one. This is analogous to regressing a variable on itself lagged once when looking for long-run dynamics, versus more lags when examining cycles (see [11]). In the second step, the number of lags is then increased to capture remaining cycles. The two models reduce the series y_t to serially uncorrelated error e_t, although the input to the trend model alone, y_t^*, is serially correlated at high frequencies. Separate trend and cycle models are combined into one stacked model:

$$\begin{aligned} \begin{pmatrix} \tau_{t+1|t} \\ \eta_{t+1|t} \end{pmatrix} &= \begin{pmatrix} A_\tau & B_\tau \, C_\eta \\ 0 & A_\eta \end{pmatrix} \begin{pmatrix} \tau_{t|t-1} \\ \eta_{t|t-1} \end{pmatrix} + \begin{pmatrix} B_\tau \\ B_\eta \end{pmatrix} e_t, \\ y_t &= \begin{pmatrix} C_\tau & C_\eta \end{pmatrix} \begin{pmatrix} \tau_{t|t-1} \\ \eta_{t|t-1} \end{pmatrix} + e_t, \end{aligned} \tag{21}$$

which constitutes a state-space model in its own right. This stacked model can be solved for forecasts of the original series regardless of the specific details of particular trend and cycle models.

If one views m time series as cycling in the short run around a stable long run equilibrium, the procedure based on the error correction specification is appropriate. In this case, cycle model depends on trend model, but not the reverse. The algorithm requires the estimation of the parameters of the cycle model identical in form to (19) first, where y_t^* now represents the long run equilibrium value, and $\tau_{t|t-1}$ is the state vector associated with higher frequencies. The serial correlation due to slow dynamics was purposely left in error y_t^*, which then serves as an output of another state-space model identical in form to (20), designed to capture the remaining trends. Separate cycle and trend models are combined into one stacked model like in (21), which can be solved for forecasts of the original series. To separate dynamic effects empirically according to the error correction model, the number of lags in the Hankel matrix should be set fairly large to capture cycles first, and then very much smaller in the second step to model the remaining trends.

3.2. Deterministic Nonstationarities

A different class of nonstationarity arises when the process depends on exogenous determinants that may not even be stochastic. Examples are deterministic trends or cycles, dummy variables for particular events or any other nonstationary exogenous (known and observable) variables. One approach to incorporate these effects requires the stochastic (innovations) model (1) be augmented with the regression equation. The procedure will remove time varying deterministic mean, thus rendering the resulting series stationary. Stationary residuals from the first step are then modeled as a stochastic process in the second step.

In the alternative approach introduced in this paper, an original time series (scalar or vector-valued) is first modeled as an impulse response of a linear time-invariant discrete system (i.e., the response of a linear system to a discrete time Kronecker delta function) observed in noise. In state-space description, such a system is specified as

$$\begin{aligned} \chi_{t+1|t} &= F\chi_{t|t-1} + b\delta_t, \\ h_t &= G\chi_{t|t-1}, \end{aligned} \tag{22}$$

where F, b, and G are parameters to be estimated, χ_t is a vector of unobservable states, and δ_t is Kronecker's delta function defined earlier. Observations on time series $\{y_t\}$ are assumed to be given by the impulse response sequence observed in noise $y_t = h_t + u_t$; $t = 1, \dots, T$, with $\{u_t\}$ being a sequence of zero mean, mutually independent Gaussian random variables.

The method is based on the fundamental result of the linear systems theory [12, pp. 164–165] that the z-transform of the output signal equals the product of the system transfer function and the z-transform of the input signal, i.e., $Y(z) = H(z)\,U(z)$. In particular, when input is defined as the impulse (Kronecker delta) function, since $U(z) = 1$, the z-transform of the systems's impulse response equals the system transfer function, i.e., $Y(z) = H(z)$. Consequently, the inverse z-transform of the transfer function $H(z)$ will return the output signal itself, and hence, model the original time series $\{y_t\}$.

Since the impulse response of the linear system is equal to the sequence of Markov parameters [13, pp. 92–93]

$$h_t = GF^{t-1}b, \qquad t \geq 1, \tag{23}$$

which are uniquely determined by inverse z-transforming the transfer function

$$H(z) = \sum_{i=1}^{\infty} h_i\, z^{-1} = \sum_{i=1}^{\infty} (GF^{i-1}b)\, z^{-1}, \tag{24}$$

effectively modeling time series $\{y_t\}$ reduces to the estimation of the parameters F, b, and G that generate Markov sequence $\{h_t\}$.

An operational procedure calls for the construction of the $(M \times N)$ Hankel matrix from $(m \times 1)$ block elements of individual observations on $\{y_t\}$ as

$$\hat{H} = \begin{pmatrix} y_1 & y_2 & \cdots & y_N \\ y_2 & y_3 & \cdots & y_{N+1} \\ \vdots & \vdots & \ddots & \vdots \\ y_M & y_{M+1} & \cdots & y_T \end{pmatrix}, \tag{25}$$

where $(M + N - 1)$ equals the total number of observations T. Exact specification of the Hankel matrix (25) is determined by using the result from [14]. They showed that the variance of errors in the frequency estimate of a single sinusoid in the presence of additive noise has a broad minimum for $M \approx 2T/3$. If $\{h_t\}$ are Markov parameters of the matrix transfer function $H(z)$ from (24), then the minimal order $\tilde{n}$ of any state-space realization of $H(z)$ is given by the rank of the Hankel matrix [13, p. 442]. The order of the model $\tilde{n}$, determined by the SVD of the Hankel matrix as the number of significant singular values, equals the dimension of the F matrix whose eigenvalues correspond to the poles of the transfer function.

The location of poles of the transfer function allows characterization of the response properties of linear systems. For example, when the transfer function has a pair of complex-conjugate poles given by $p_1 = a + ib$ and $p_2 = a - ib$, such a pair of poles will give rise to the response term $\beta r^k \cos[k\theta + \alpha]$ for $k = 0, 1, 2, \ldots$, where $r = \sqrt{(a^2 + b^2)}$ is the magnitude of the pole, $\theta = \tan^{-1}(b/a)$ is its angle, and β and α are constants obtained from the partial-fraction expansion of $Y(z)$. If $\sqrt{(a^2 + b^2)} > 1$, poles are located outside the unit circle (an unstable system) which results in a sinusoidal-like oscillation increasing in magnitude. If poles are located on the unit circle, the response would be a constant sinusoidal oscillation. Finally, if poles are located inside the unit circle (stable system), the response would be sinusoidal oscillation decreasing in amplitude [12, p. 260].

After the Hankel matrix (25) has been specified, the rest of the procedure for estimating parameters of the impulse response model closely follows the estimation of the covariance model parameters from Section 2.1. Estimates of F and G are derived from the following property of the observability matrix:

$$\begin{pmatrix} G \\ GF \\ \vdots \\ GF^{M-2} \end{pmatrix} (F) = \begin{pmatrix} GF \\ GF^2 \\ \vdots \\ GF^{M-1} \end{pmatrix}, \tag{26}$$

where the $((M \times m) \times \tilde{n})$ observability matrix resulting from the factorization of the Hankel matrix (25) is partitioned into two submatrices: V_1 containing the first $((M - 1) \times m)$ rows, i.e., from row 1 to $((M - 1) \times m)$, and V_2 containing the last $((M - 1) \times m)$ rows, i.e., from row $(m + 1)$ to $(M \times m)$. By simple inspection, the estimate of matrix G is simply the first block row of V_1, and the estimate of F is obtained by rewriting (26) as $V_1 F \approx V_2$ and solving this over-determined system in the least square sense:

$$\hat{F} = (V_1' V_1)^{-1} V_1' V_2. \tag{27}$$

To estimate b, notice that the last element in the Hankel matrix (25) is $y_T = h_T + u_T$, where $h_T = GF^{M+N-2} b$ is the final term in the sequence of Markov parameters. Having this in mind, the following relation holds

$$\begin{pmatrix} G \\ GF \\ \vdots \\ GF^{T-1} \end{pmatrix} b \approx \begin{pmatrix} y_1 \\ y_2 \\ \vdots \\ y_T \end{pmatrix}. \tag{28}$$

Denoting the new observability matrix in (28) by W, and the RHS vector consisting of $(m \times 1)$ block elements of the individual observations on time series $\{y_t\}$ stacked vertically by $\mathbf{y}$, the system can be solved in the least square sense to give the estimate of b:

$$\hat{b} = (W' W)^{-1} W' \mathbf{y}. \tag{29}$$

The dimension of W is $((T \times m) \times \tilde{n})$, where m is the number of series jointly modeled, and the $((T \times m) \times 1)$ vector $\mathbf{y}$ consists of $(m \times 1)$ block elements of the individual observations on time series $\{y_t\}$ stacked vertically. Therefore, in the deterministic model of (22), b is a $(\tilde{n} \times 1)$ vector, while B from the innovations model (1) is a $(\tilde{n} \times m)$ matrix.

With estimated parameters F, b, and G, the forecasts of the original time series are generated and forecasting errors (residuals) are calculated. In the second step, the innovation model (1) is used to model errors from the first step. The forecasts of the original time series are obtained by summing the forecasts from both steps.

4. JAPANESE WHOLESALE SALMON MARKET

Japan is the largest export market for United States fishery products. In 1990 Japan imported approximately 498,850 MT of edible fishery products, valued at $1.79 billion [15]. The salmon industry, the largest finfish fishery in the United States, is heavily dependent on this market. In the latter half of the 1980's, 79% to 89% of the total supply of fresh/frozen salmon was exported. Of those fresh/frozen salmon exports, in 1989, 79% or 118,600 MT, valued at $627.5 million, was exported to Japan [16]. Despite the dependence the U.S. salmon industry has on Japan, surprisingly little work has been undertaken to understand how this market behaves.

In 1990 Japan consumed over one-third (422,000 MT) of the world's supply of salmon [17]. Imports, which were insignificant in 1976, accounted in 1990 for nearly 40% (161,000 MT) of Japan's total supply. The remainder of Japan's supply is derived from a domestic hatchery-based ocean fishery consisting primarily of chum salmon, limited sea harvest, and domestic coho aquaculture. The U.S. and Canada are the primary foreign suppliers for Japan's frozen salmon, with 72% and 12% of frozen salmon imports respectively, in 1990.

The dominant price determining mechanism in Japan consists of 54 central wholesale markets and 72 local wholesale markets throughout the country. The largest central wholesale market is Tsukiji Central Wholesale Market in Tokyo. This market represents a highly competitive environment in which frozen salmon prices are determined on a daily basis. These prices then become reference prices for outside-the-market transactions (see [18]). Salmon traders in Japan use both private and public information in price determination. Publicly available information widely reported in Japanese trade press includes monthly quantities sold, monthly inventory holding, and the annual forecast of Bristol Bay, Alaska sockeye salmon harvest (the largest sockeye fishery in the world) reported by the Alaska Department of Fish and Game [19].

The prices determined in Japan are closely tied to wholesale and ex-vessel prices paid in Alaska. Arrangements vary from firm to firm, but there is generally a formula used by firms to derive reasonable wholesale prices in Alaska based on Japan's prices minus commissions, transportation, insurance, and other conversions. Clearly, improved knowledge of price behavior in Japan could be valuable to U.S. and Japanese salmon processors and traders in determining what prices to pay fishermen and whether or not to accumulate, hold, or sell frozen salmon throughout the year.

In this paper, we explore the price dynamics of the two most important species of salmon sold in Japan, sockeye and chum. The objective is to simultaneously model the wholesale prices of five species/products as a vector-valued time series. These include: frozen chum, salted chum, salted fall chum, frozen sockeye, and salted sockeye. The observations are the average wholesale prices of the three wholesale markets in the Tokyo area as reported by the Tokyo Municipal Government [20]. The data set covers the period from 1978 to 1991, for the total of 168 monthly observations.

The stationarity of time series is usually tested by the Dickey-Fuller test. Although it is widely used, its power is limited because it only tests for the presence of stochastic trends (random walks), while nothing can be said about other possible forms of nonstationarity. In case of the salmon market, there is evidence that price series may not be stationary due to visible presence of cycles. Cycles are determined by complex interplay of various factors, such as fish population dynamics, seasonal variations in demand and cyclical nature of overall business activity. In order to test whether any of the wholesale salmon prices can be best described as a random walk process, perhaps with trend, a Dickey-Fuller unit root test was run for each of the price series individually. The calculated F-ratios are 4.08, 5.65, 8.91, 3.90, and 5.51, respectively, and the rejection of the hypothesis of a random walk failed in all cases, except in case of the salted fall chum price.[1] These results, together with the cyclical nature of the salmon market, suggest

[1]The critical value for Dickey-Fuller statistics at the 5% level of significance, and 100 to 250 observations is between 6.49 and 6.34 (see [21, pp. 459–465]).

Table 1. Japanese wholesale salmon market: stochastic nonstationarities removal procedure.

Parameter Estimates	Cointegration Model: $obs = 162$, $m = 5$				
Trend: $k = 1$, $n = 1$					
A	0.9251				
B	0.0124	0.0053	0.0012	0.0059	0.0052
C'	35.8308	26.4727	25.2963	35.2224	32.0347
Cycle: $k = 6$, $n = 2$					
A	0.9314	−0.0281			
	−0.0049	0.8759			
B	0.0058	−0.0059	0.0117	−0.0106	−0.0091
	−0.0009	0.0019	0.0015	0.0030	−0.0040
C'	13.8504	−7.5871	6.9335	−7.7530	−9.0617
	1.5027	35.6063	5.4543	9.2394	−44.1086
	Error Correction Model: $obs = 162$, $m = 5$				
Cycle: $k = 12$, $n = 2$					
A	0.8624	−0.2340			
	0.0720	1.0588			
B	0.0312	0.0103	0.0024	0.0043	−0.0007
	−0.0131	0.0058	0.0015	0.0021	0.0021
C'	14.8155	9.7489	10.5084	14.3666	12.4053
	5.1052	3.1409	6.3199	7.9777	6.5882
Trend: $k = 1$, $n = 2$					
A	0.8311	−0.1218			
	0.0211	0.9005			
B	0.0013	−0.0033	0.0022	−0.0049	−0.0047
	−0.0052	−0.0003	−0.0014	0.0007	−0.0002
C'	1.8596	−45.7504	10.6471	−39.8526	−50.6686
	−64.5124	−26.7182	−32.3674	9.3468	−8.1670

transformation of the original series to achieve stationarity before the standard methods can be implemented.

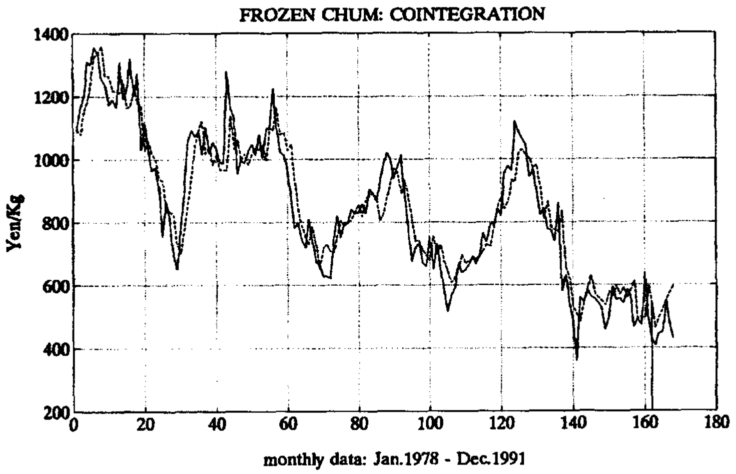
5. ESTIMATION PROCEDURES

All four models presented earlier are estimated with the first 162 data points, and in-sample forecasts are generated. The last six months of 1991 prices are saved for the out-of-sample forecasts validation.

In the cointegration model, to estimate common trends the number of lags is set to $k = 1$, and the resulting singular values of the Hankel matrix are 4.8271, 0.4992, 0.2308, 0.1279, and 0.0816. One dominant singular value suggests that prices are cointegrated with one common stochastic trend ($\tilde{n} = 1$). To model any remaining cycles in the detrended data, the number of auto-covariance lags in the Hankel matrix is increased to $k = 6$. Decaying pattern of the singular values of the new Hankel matrix shows two dominant singular values, signaling the order of the cycle model of $\tilde{n} = 2$.

In the error correction specification, after setting the number of lags to $k = 12$, which gives the most distant auto-covariance lag to be modeled of two years (24 months), the order of the cycle model turns out to be $\tilde{n} = 2$. To estimate trend model, the lag parameter is reduced to $k = 1$, and the SVD shows two dominant singular values, suggesting two common leftover trends ($\tilde{n} = 2$).

The parameter estimates of the cointegration and error correction models are presented in Table 1, and the procedures are illustrated in Figure 1, by plotting forecasts (dashed line) against actual prices (solid line) using frozen chum and frozen sockeye wholesale price series as examples. The vertical line at the 162^{nd} observation separates the in-sample from the out-of-sample forecasts.



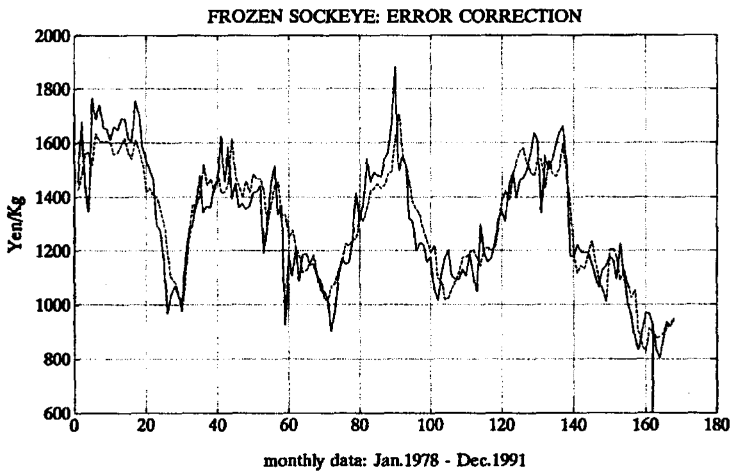


Figure 1. Forecast of Japanese wholesale salmon prices. Stochastic nonstationarities removal procedures.

In an attempt to model possible deterministic nonstationarities in Japanese wholesale salmon prices, a set of demand equations is added to the innovations model (1). The whole system has the following form:

$$\begin{aligned} x_{t+1|t} &= Ax_{t|t-1} + Be_t, \\ u_t &= Cx_{t|t-1} + e_t, \\ P_t &= \alpha + \beta Q_t + \gamma I_t + \delta D_t + u_t, \end{aligned} \tag{30}$$

where P_t is a (5×1) vector of wholesale prices for frozen chum, salted chum, salted fall chum, frozen sockeye, and salted sockeye in month t; Q_t is a (5×1) vector of the corresponding quantities sold in month t (see [20]), I_t is a (2×1) vector of explanatory variables describing the status of salmon inventories in Japan, and D_t is a (3×1) vector of quarterly dummies. The first variable in the inventory subset is defined as the outflow from salmon inventories in month t other than quantities of five species/products explicitly accounted for by the vector Q_t. The second variable is defined as the percentage increase (decrease) in total inventory level in the current month over the inventory level in the corresponding month a year ago (see [22]). The first two quarterly dummies (January–March and April–June) are multiplied by the ratio of the preseason forecasts of sockeye salmon returns to Bristol Bay to its 10-year moving average (see [19]).

The system (30) is estimated using an iterative procedure suggested in [8]. First, five demand equations are estimated by the ordinary least squares (OLS). Then, using the resulting residuals u_t as observations, the state-space model is estimated generating forecasts $\hat{u}_t$. These forecasts are then used to construct a new series $P_t^* = P_t - \hat{u}_t$. The structural parameters α and β are then reestimated using P_t^* instead of P_t, and the whole process is repeated until the parameters converge.

The original prices in yen/kg are converted to the real January 1991 prices by using the Japanese wholesale price index. The wholesale quantities are measured in metric tons. Demand equation parameters converged after three iterations. Forecasts are obtained by summing the forecasted residuals (generated by the innovations model) from each of the three iterations and adding them to the prices predicted by the last iteration regression equation. The final iteration parameter estimates are presented in Table 2, and the entire procedure is illustrated in Figure 2, using salted chum prices.

Table 2. Japanese wholesale salmon market: structural model procedure for the removal of deterministic nonstationarities.

Parameter Estimates	Frozen Chum	Salted Chum	Salted Fall Chum	Frozen Sockeye	Salted Sockeye
	Structural Model				
Final (third) iteration					
Adj. R^2	0.64680	0.57850	0.60040	0.63100	0.62400
DW	1.59860	1.78840	1.70860	1.72230	1.85020
α	1431.0	1894.9	1077.7	1915.2	2055.5
β	$\boxed{0.05266}$	0.01152	0.06525	0.02388	0.02051
	0.11184	$\boxed{0.04247}$	0.05776	0.12017	0.12295
	0.03631	0.04084	$\boxed{0.03765}$	0.02974	0.03487
	−0.04678	−0.01530	−0.01185	$\boxed{-0.03304}$	−0.00435
	0.02540	−0.22648	−0.22190	−0.48475	$\boxed{-0.45549}$
γ	−0.02788	−0.02045	−0.01705	−0.02078	−0.02036
	−1.14880	−1.02330	−0.28213	−1.19220	−1.20380
δ	−1.6085	−1.5307	−1.3548	−1.5356	−1.4922
	−1.5606	−1.6917	−1.1211	−1.5376	−1.7891
	−86.822	−196.610	−72.871	−97.304	−140.710
	Innovations Model: $obs = 162$, $m = 5$, $k = 2$, $n = 2$				
Final (third) iteration					
A	0.7899	0.2912			
	0.0243	0.6679			
B	0.0033	−0.0021	0.0004	0.0003	0.0014
	−0.0016	0.0009	0.0011	−0.0014	0.0019
C'	45.6260	22.6582	37.6553	29.8285	17.2058
	−96.0695	49.3347	−45.4971	−29.3749	29.5746

An alternative technique for removing deterministic nonstationarities replaces the structural model by an impulse response model. To avoid the possibility of high frequency cycles being swamped by long swings of the trend, the empirical procedure starts with detrending of the

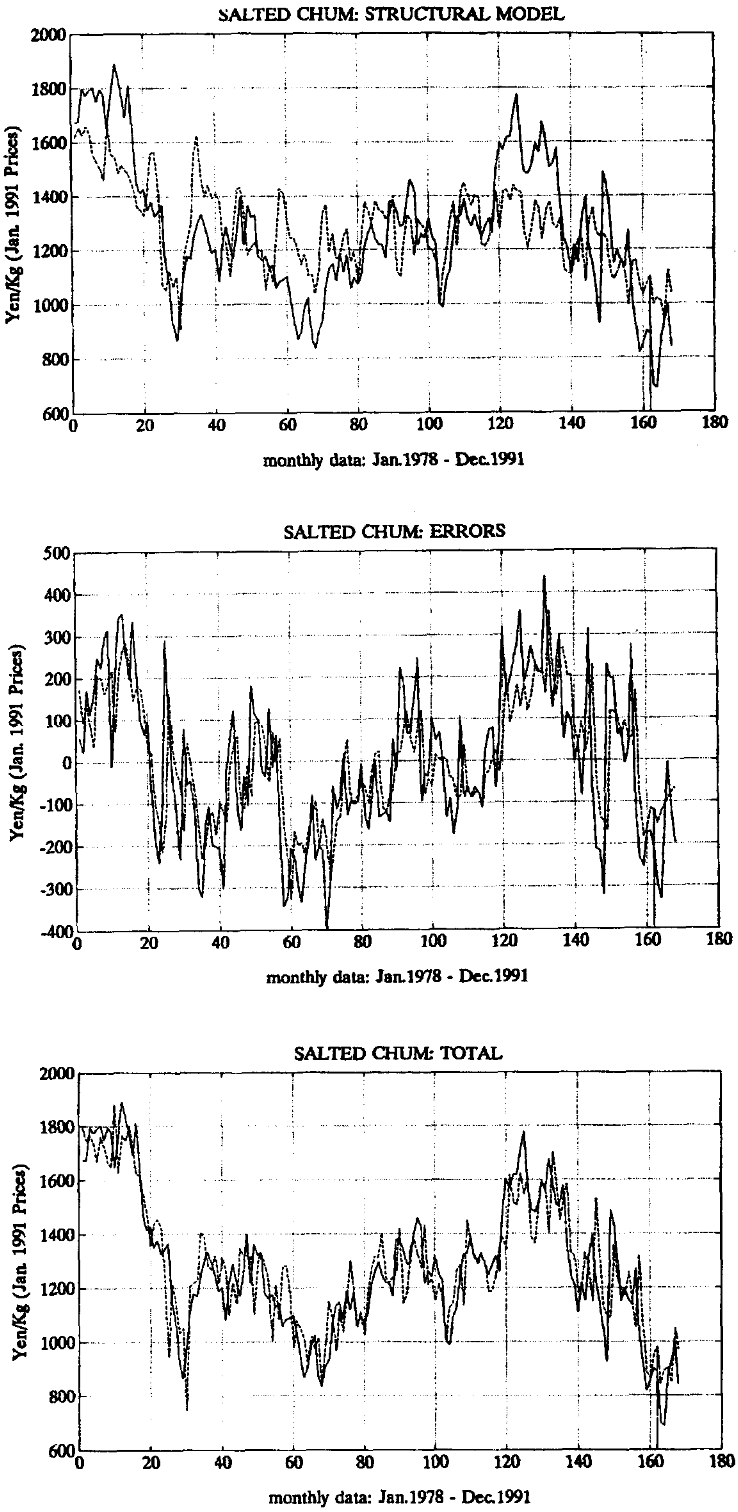


Figure 2. Forecasts of Japanese wholesale salmon prices. Structural model procedure for the removal of deterministic nonstationarities.

original time series. This is done by fitting the linear time trend through the vertically stacked original data using five dummy variables for five species/products.[2] The idea is that the entire

[2]Modeling original data without prior detrending is also possible. In this case, the estimated rank of the Hankel matrix, i.e., the order of the model, equals 4, and the eigenvalue decomposition of the matrix A gives one conjugate-

market (all five products) is driven by the common dynamics, and the constant price differences (intercepts) reveal perceived quality differences.

Using detrended data, the Hankel matrix (25) is formed, and its rank is estimated by the SVD as 2. The eigenvalue decomposition of the matrix F gives the complex-conjugate pair of eigenvalues: $0.9897+0.1486i$ and $0.9897-0.1486i$ with moduli of $r = 1.0008$ (indicating a slightly unstable system), and the angle of $\theta = \pm 0.149$. The frequency estimate calculated as $\theta/2\pi$ yields 0.0237 cycles per month (0.2845 cycles per year), or equivalently the period of 3.5145 years for one full cycle. Parameter estimates of the impulse response model together with those of the innovations model are presented in Table 3, and the procedure is illustrated in Figure 3 using salted sockeye as an example.

Table 3. Japanese wholesale salmon market: impulse response model procedure for the removal of deterministic nonstationarities.

Parameter Estimates	Impulse Response Model: $obs = 162$, $m = 5$				
Trend					
time	−2.1683				
intercepts	1034.8	1481.3	860.9	1492.3	1665.9
Cycle: $n = 2$					
A	0.9896	−0.1626			
	0.1357	0.9898			
B	2051.2				
	−388.5				
C'	0.0393	0.0402	0.0149	0.0517	0.0532
	0.0610	0.0644	0.0499	0.0898	0.0629
	Innovations Model: $obs = 162$, $m = 5$, $k = 3$, $n = 3$				
A	0.8395	−0.0192	1.4450		
	0.0182	0.9337	−0.1911		
	−0.0042	0.0146	0.7753		
B	0.0229	0.0125	0.0077	0.0145	0.0109
	−0.0088	−0.0012	−0.0006	0.0016	0.0020
	0.0002	0.0029	−0.0001	0.0016	−0.0025
C'	14.9886	19.0726	11.4056	20.7136	18.1862
	−46.1846	25.1393	−15.1170	10.8555	25.4102
	−25.5013	52.7562	−45.0715	−23.2435	−106.7566

6. SUMMARY OF FORECASTS AND CONCLUSIONS

In order to compare alternative models, it is necessary to have a criterion by which the performance of the predictors is measured. The usual criterion is mean square error (MSE). Its advantage is that it can be directly compared to the zero lag auto-covariance matrix Γ_0 to see by how much the modeling procedure has reduced the unconditional error sum of squares. To compare the models of different processes (time series), the mean absolute percentage error (MAPE) can also be used. MAPE measures the average percentage deviation of forecast from its corresponding actual value, and is therefore dimensionless. The model which generates forecasts with smallest statistics is according to these criteria the most favorable one. Results of comparing different estimation and forecasting procedures are summarized in Table 4.

Using MSE criterion, the procedure that combines impulse response model with innovations model (#3) consistently gives the best results. In case of frozen chum prices, for example, the

complex pair and two real eigenvalues very close to unity. Two real eigenvalues of unit magnitude model the linear trend (intercept and slope), and one conjugate-complex pair models a cycle.

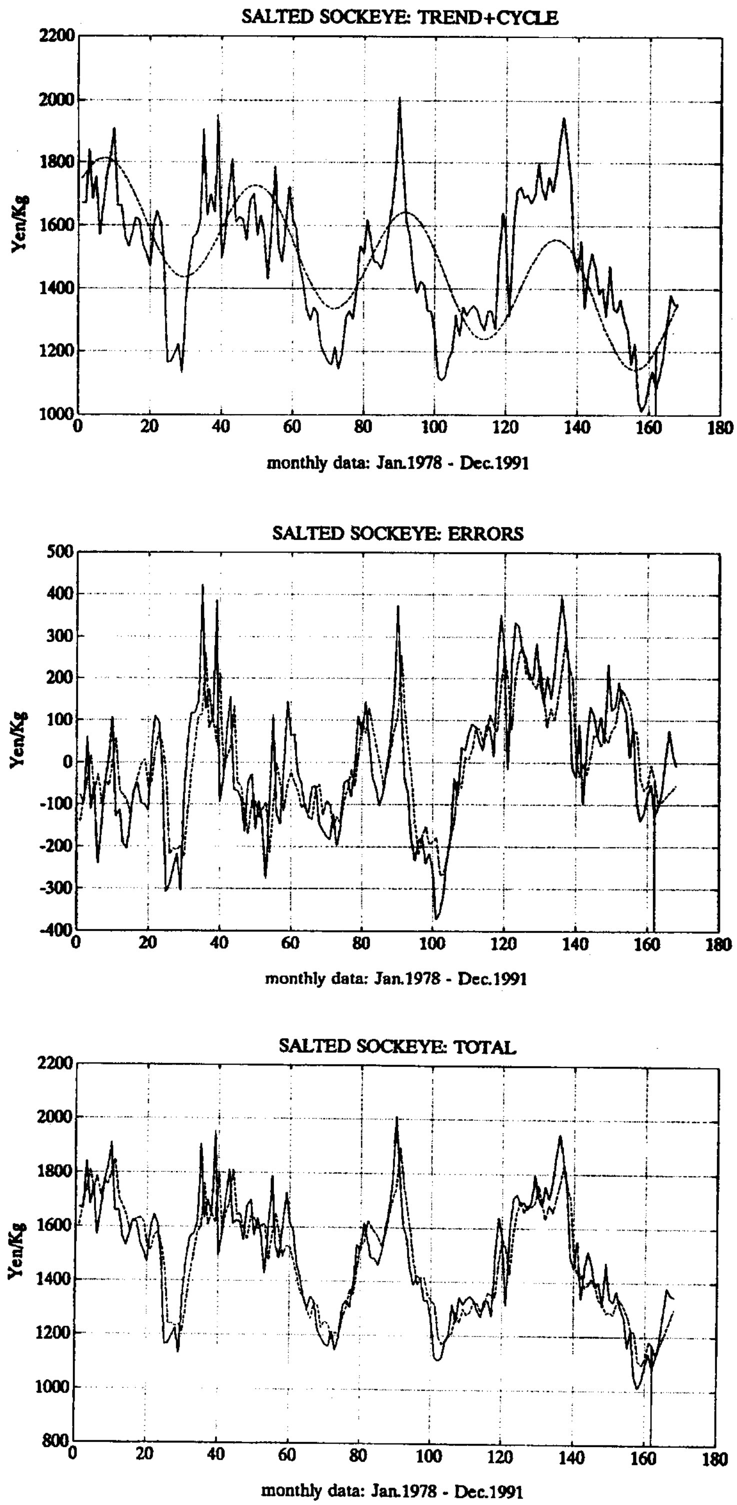


Figure 3. Forecasts of Japanese wholesale salmon prices. Impulse response model procedure for the removal of deterministic nonstationarities.

model reduces the unconditional error sum of squares (Γ_0) by almost ten times. Using MAPE criterion, forecasts of a fairly similar quality are obtained by cointegration model (#1), error correction model (#2), and impulse response model (#3). Average deviations of forecasts from their corresponding actual values for all species/products are between 6% and 10%. Forecasting per-

Table 4. Japanese wholesale salmon market: price forecasting results.

Species/Product	Frozen Chum	Salted Chum	Salted Fall Chum	Frozen Sockeye	Salted Sockeye
Comparison of In-Sample Forecasting Statistics: *obs* = 162					
Γ_0	5408.8	3398.1	2918.6	5095.0	4734.6
Stochastic Nonstationarities					
1. Cointegration Model					
MSE	664.9	1110.7	753.1	1049.1	1176.3
MAPE	7.86%	6.70%	10.26%	6.03%	5.57%
2. Error Correction Model					
MSE	615.4	1199.1	759.3	1109.5	1405.3
MAPE	7.22%	6.79%	10.66%	6.41%	6.11%
Deterministic Nonstationarities					
3. Impulse Response Model					
MSE	599.3	1013.7	755.7	1107.6	1114.9
MAPE	7.53%	6.13%	10.31%	6.07%	5.46%
4. Structural Model					
MSE	1354.7	1594.0	1011.4	1659.3	1608.7
MAPE	12.32%	7.70%	12.19%	8.14%	6.77%
Best Out-Of-Sample Forecasting Results for 1991					
Best Model	2	4	3	2	3
MAPE	6.51%	13.87%	12.40%	3.95%	5.31%
Actual Prices (yen/kg)					
June	429	883	294	930	1081
July	408	693	344	836	1122
August	444	681	326	804	1180
September	450	869	383	884	1287
October	551	918	487	937	1381
November	477	981	569	919	1354
December	430	828	550	948	1346
Forecasts (yen/kg)					
July	413	836	323	876	1129
August	419	886	345	883	1161
September	430	894	369	895	1196
October	446	835	392	910	1232
November	466	1033	416	929	1267
December	490	956	440	950	1301
Errors (yen/kg)					
July	−5	−143	21	−40	−7
August	25	−205	−19	−79	19
September	20	−25	15	−11	91
October	105	83	95	27	150
November	11	−52	153	−10	87
December	−60	−267	110	−2	45

formance of the procedure that combines regression equation (structural model) with innovations model (#4) is, according to both criteria, somewhat inferior to the other three techniques.

Using the parameters estimated with the first 162 observations, six-months-ahead out-of-sample forecasts are generated without iteratively updating state vectors. Hence, these are truly out-of-sample forecasts with June 1991 as the last observation effectively used. For each individual species, we present only the result of the best forecasting procedure (forecasts with smallest errors). Using MAPE criterion, error correction model (#2) works the best for frozen chum and frozen sockeye, impulse response model (#3) is the best model for salted fall chum and salted sockeye, and structural model (#4) produces the best forecasts in the case of salted chum. The cointegration model that works nicely for in-sample forecasting is always outperformed by at least one other model in out-of-sample forecasting. The overall best out-of-sample forecasts are obtained by the error correction procedure in case of frozen sockeye prices where forecasting errors range from 2 to 79 yen/kg. Using the exchange rate of 130 yen/$, these errors range from 1.5 to 61 cents/kg.

In many instances, forecast directions are more important than magnitudes. Taking June 1991 as a reference point to which all future months are compared to determine the market direction, the best results are obtained with the error correction model (salted fall chum and salted sockeye) where 100% of the signals (directions) are correct. The impulse response model (frozen chum and frozen sockeye) has generated 83% (5/6) correct signals, and structural model (salted chum) 33% (2/6). By analyzing market directions on the month-to-month basis (current month vs. previous month), the results are identical for all four procedures, with 67% (4/6) correct signals (directions).

We have modeled prices of the group of five salmon species/products on the Tokyo wholesale market using four state-space techniques for modeling vector-valued nonstationary time series. When tested for the presence of stochastic trends, four out of five series appeared to follow random walk, and when plotted against time, all price series indicate pronounced cyclical behavior. Although salmon prices are very difficult to predict, presented techniques produced a set of six-steps-ahead forecasts that are surprisingly good in terms of both magnitude of their errors and percentage of correct signals, encouraging future research in the area.

REFERENCES

1. M. Aoki, *State Space Modeling of Time Series*, Springer-Verlag, Berlin, (1987).
2. M. Aoki, *State Space Modeling of Time Series*, 2$^{\text{nd}}$ edition, Springer-Verlag, Berlin, (1990).
3. A. Havenner and M. Aoki, Deterministic and stochastic trends in state space models of nonstationary time series, Working Paper 88-17, Department of Agricultural Economics, University of California at Davis, Davis, CA, (1988).
4. M. Aoki and A. Havenner, State space modeling of multiple time series, Working Paper 89-9, Department of Agricultural Economics, University of California at Davis, Davis, CA, (1989).
5. S.Y. Kung, A new identification and model reduction algorithm via singular value decomposition, In *Proceedings of the* 12$^{\text{th}}$ *Asilomar Conference*, Pacific Grove, CA, (1978).
6. R.J. Vaccaro, Finite-data algorithms for approximate stochastic realization, In *Modelling and Application of Stochastic Processes*, (Edited by U.B. Desai), Kluwer Academic Publishers, Boston, (1986).
7. R.J. Vaccaro and T. Vukina, A solution to the positivity problem in the state-space approach to modeling vector-valued time series, *Journal of Economic Dynamics and Control* **17**, 401–421 (1993).
8. J.H. Dorfman and A. Havenner, State space modeling of cyclical supply, seasonal demand, and agricultural inventories, *American Journal of Agricultural Economics* **73**, 829–840 (1991).
9. M. Aoki, Cointegration, error correction, and aggregation in dynamic models: A comment, *Oxford Bulletin of Economics and Statistics* **50**, 89–95 (1988).
10. R.F. Engle and C.W.J. Granger, Cointegration and error correction: Representation, estimation, and testing, *Econometrica* **55**, 251–276 (1987).
11. M. Cerchi and A. Havenner, Cointegration and stock prices: The random walk on Wall Street revisited, *Journal of Economic Dynamics and Control* **12**, 333–346 (1988).
12. J.A. Cadzow, *Discrete Time Systems: An Introduction with Interdisciplinary Applications*, Prentice-Hall, Inc., Englewood Cliffs, NJ, (1973).
13. T. Kailath, *Linear System Theory*, Prentice-Hall, Inc., Englewood Cliffs, NJ, (1980).
14. A.C. Kot, S. Parthasarathy, D.W. Tufts and R.J. Vaccaro, Statistical performance of single sinusoid frequency estimation in white noise using state-variable balancing and linear prediction, *IEEE Transactions on Acoustics, Speech, and Signal Processing* **ASSP-35** (11), 1639–1642 (1987).
15. U.S. Dept. of Commerce, *Fisheries Statistics of the U.S.*, NOAA, NMFS, Silver Spring, MD, (1991).
16. G. Knapp and T. Smith, *The Alaska Seafood Industry*, Institute of Social and Economic Research, University of Alaska, (1991); Prepared for Alaska Dept. of Commerce and Economic Dev. and the Alaska Export Authority, Juneau, AK.

17. C. Becker, Market advisory: Salmon trends in north Japan, U.S. Dept. of Commerce, International Trade Commission, Anchorage, AK, (1991).
18. Y. Kusakabe and J.L. Anderson, Analysis of the Japanese salmon market, Report 25/89, Center for Applied Research, Norwegian School of Economics and Business Administration, Bergen, Norway, (1990).
19. Preliminary Alaska commercial fisheries harvests and values, Regional Information Report, various issues, Alaska Department of Fish and Game, Juneau, AK.
20. *Tokyo Central Wholesale Market Yearbook*, (in Japanese), various years and pages, Tokyo Municipal Government.
21. R.S. Pindyck and D.L. Rubinfeld, *Econometric Models and Economic Forecasts*, 3rd edition, McGraw-Hill, New York, (1991).
22. *Aquatic Products Distribution Statistical Yearbook*, various issues (in Japanese), Ministry of Agriculture, Forestry and Fisheries, Department of Statistical Information.